

ERIC Document Resume		See Instruction Sheet		DATE OF RESUME 7/1/65		DATE RECEIVED	
A. PUBLICATION DATA							
1. ERIC NUMBER ED002134		2. ORIGINATOR CONTROL NO(S). N.A.		3. IDENTIFICATION NO. N.A.		DO NOT WRITE IN SPACE BELOW	
4. AUTHOR(S): Brant, Vincent and Gerardi, William						MICROFILM COPY AVAILABLE YES ☒ NO ☐	
5. TITLE OF DOCUMENT ABSTRACTED: A Tentative Guide: Differential and Integral Calculus						COPYRIGHT GRANTED YES ☐ NO ☒	
6. PUBLICATION TITLE: N.A.						COPYRIGHT RELEASE FILED YES ☐ NO ☐	
7. EDITOR(S): N.A.				8. PLACE OF ORIGIN Baltimore County Public ORG. Schools, ST. Towson 4, Maryland CITY, STATE		TYPE OF REPORT PROPOSAL ☐ IN-PROGRESS ☒ FINAL ☐ FOLLOW-UP ☐	
9. PUBLISHER OR SOURCE Baltimore County Public Schools, Towson 4, Maryland						OTHER ERIC REFERENCES:	
10. VOL. N.A.	11. PAGES N.A.	12. MO./YR. 9/63	13. TOTAL PP. 61	TELE. INST/LOC CODE	EXT.	ERIC SATELLITE CODE	
B. INDEXING DATA							
14. SUBJECT AREA				15. TYPE OF ENDEAVOR Teachers' Guide			
16. INDEXING TERMS (6-12)							
curriculum planning		secondary education					
calculus instruction		advanced students		mathematics			
analytic geometry		gifted students		superior students			
17. ABSTRACT (MAX. 250 WORDS)							
The course is intended to go beyond the requirements of the Advanced Placement Program in Mathematics as designed by the College Entrance Examination Board. The Advanced Placement Program consists of a one year course combining analytic geometry and calculus. Presupposed here are: a semester course in analytic geometry and a thorough knowledge of algebra, analytic trigonometry, and elementary analysis. A study of functions, including polynomial, power, exponential, logarithmic, circular, and inverse circular functions is desirable. The course is designed for those students who desire to major in mathematical or scientific fields. Students selected to participate should be characterized by high mathematical ability, interest, and excellent performance. Units include the rate of change of a function, derivatives of algebraic functions, integration, transcendental functions, techniques of integration, polar coordinates, plane analytic geometry vectors and parametric equations, and solid geometry and vectors.							
ERIC No. 18. SOURCE OF ABSTRACT							
C. FUNDING DATA							
19. SUPPORTING ORGANIZATION				20. DIVISION OR DEPT.		21. CONTRACT NO.	
						PROJECT NO.	
				22. TOTAL COST OF PROJECT			
				23. DURATION OR PROJECT		U.S. Department of Health, Education, and Welfare Office of Education	

ED0062134

A TENTATIVE GUIDE

**DIFFERENTIAL
AND
INTEGRAL CALCULUS**

BALTIMORE COUNTY PUBLIC SCHOOLS
1963

**REPRINTED BY PERMISSION
U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
OFFICE OF EDUCATION**

THIS DOCUMENT HAS BEEN PRINTED EXACTLY AS RECEIVED FROM THE PERSON
OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT
NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

BALTIMORE COUNTY PUBLIC SCHOOLS

A Tentative Curriculum Guide

DIFFERENTIAL AND INTEGRAL CALCULUS

Committee

Vincent Brant
William Gerardi

Towson 4, Maryland
September 1963

TABLE OF CONTENTS

BASIC CONSIDERATIONS AND SPECIFIC SUGGESTIONS	1
MATERIALS OF THE COURSE IN CALCULUS	2
UNIT ONE - THE RATE OF CHANGE OF A FUNCTION	3
UNIT TWO - DERIVATIVES OF ALGEBRAIC FUNCTIONS	11
UNIT THREE - APPLICATIONS	17
UNIT FOUR - INTEGRATION	22
UNIT FIVE - TRANSCENDTAL FUNCTIONS	30
UNIT SIX - TECHNIQUES OF INTEGRATION (PART I)	36
UNIT SEVEN - POLAR COORDINATES	41
UNIT EIGHT - APPLICATIONS	44
UNIT NINE - PLANE ANALYTIC GEOMETRY	49
UNIT TEN - VECTORS AND PARAMETRIC EQUATIONS	55
(OPTIONAL)	
UNIT ELEVEN - SOLID GEOMETRY AND VECTORS (OPTIONAL) ...	57

BASIC CONSIDERATIONS

This curriculum guide presents a one year course in differential and integral calculus. Thus it goes beyond the requirements of the Advanced Placement Program in Mathematics as designed by the College Entrance Examination Board. The Advanced Placement Program consists of a one year course combining analytic geometry and calculus. This curriculum guide presupposes a semester course in analytic geometry, a thorough knowledge of algebra, analytic trigonometry, and elementary analysis. Furthermore, a study of functions, including polynomial, power, exponential, logarithmic, circular, and inverse circular functions is desirable.

This course in calculus is designed for those students who desire to major in mathematical or scientific fields. Students selected for this course should be characterized by high mathematical ability, interest, and excellent performance. Teachers should encourage these students to take the Advanced Placement Program Examination in mathematics which is administered by the College Entrance Examination Board about May of each year. This curriculum guide anticipates this examination by including a two-week review prior to the examination.

SPECIFIC SUGGESTIONS

The suggested assignments should be worked out by each instructor before the assignment is made. This should be done so that difficulties students may encounter can be anticipated and appropriate hints or cautions given. Some assignments may be too long. It is suggested that each instructor time himself as he does the assignment and then make appropriate adjustments for the students.

Each assignment contains both new and review work. The review is included for several reasons:

1. Students rarely, if ever, gain complete understanding of a topic after only one period of instruction.
2. A student learns best if he has constant review, repetition, and reinforcement.
3. A student will retain concepts and skills longer if he is required to participate in a planned maintenance program.

Each teacher is asked to record appropriate comments in the spaces left between topic headings. Such comments should pertain to length of assignments, sequence of topics, recommendation for other references or any ideas which would strengthen the outline. These curriculum guides will be collected at a later date so that modifications suggested by the teacher may be considered.

MATERIALS OF THE COURSE IN DIFFERENTIAL AND INTEGRAL CALCULUS

<u>Code</u>	<u>Name of Reference</u>
1	- Thomas, G. B. <u>Calculus and Analytic Geometry</u> , <u>Third Edition</u> . Addison-Wesley Publishing Co., Inc. 1960.
2	- Thomas, G. B. <u>Limits</u> . Addison-Wesley Publishing Co., Inc. 1963.
3	- Morrill, W.K. <u>Calculus</u> . D. Van Nostrand. 1956.
4	- TEMAC. <u>Introductory Calculus</u> . Encyclopedia Brittanica Films. 1960.

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT ONE THE RATE OF CHANGE OF A FUNCTION (Time allotment - 17 days)			<u>Code</u> N - New Material R - Review Material
1	Introduction (1-1) Coordinates (1-2)	1 ... 1 ...	1, 2 3, 5	N-p. 4:2, 4, 12; p. 5:19, 20
2	Directed line segments (1-3) Slope of a straight line (1-4)	1 ... 1 ...	5- 7 7-12	N-p. 7:3, 4, 11ab; p. 12:4, 13, 17 R-p. 4:3, 16
3	Slope of a straight line (1-5) Equations of a straight line (1-5)	1 ... 1 ...	7-12 13-16	N-p. 12:21; p. 13:23, 28 p. 16:5, 15, 21, 26 R-p. 5:17; p. 7:10, 11c

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
4, 5	Functions and graphs (1-6) a. Definition of function b. Domain; range c. Absolute value d. Open interval : $x - a < b$ e. Closed interval : $x - a \leq b$ f. Greatest integer function	1 ...	16-25	N-p. 25:2; p. 26:4, 8, 9ac R-p. 12:15; p. 13:22; p. 16:22 N-p. 25:1; p. 26:5, 6, 7, 9bd, 12 R-p. 16:19, 23, 24
6	Slope of a curve (1-7)	1 ...	26-28	N-p. 28:2, 8; p. 29:11 R-p. 26:3, 10; p. 16:25 Morrill p. 18:1e
7	Derivative of a function (1-8)	1 ...	29-33	N-p. 33:1, 3, 4, 5 R-p. 28:9; p. 29:14; p. 26:11 Morrill p. 18:1f

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
8	Derivative of a function (1-8)	1 ...	29-33	N-p. 33: 6, 8, 11, 15; p. 34: 18 R-p. 28: 10; p. 16: 16, 20 Morrill p. 18: 1g
9	Velocity and rates (1-9)	1 ...	34-39	N-p. 38: 2, 4, 7, 11 R-p. 33: 7, 12, 16; p. 34: 19 Morrill p. 18: 1h
10	Review (Student boardwork)	1 ...	1-39	<u>Review During Class</u> R-p. 28: 6; p. 38: 1, 3; p. 39: 13, 14; p. 33: 14; p. 34: 17, 21

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
11	Test			R-p. 50:1-5 R-p. 51:17
12	Introduction to limits (Reference 2) Note: An introduction to limits may be approached through the supplementary mimeographed material "Concept of Limit."	2 ...	1-6	N-(2) p. 3:1, 2; p. 6:1 R-Selected review problems to provide remedial work for difficulties shown on test. p. 50-52
13	Graphs of Sequences Properties of limits (1-10) Note: p. 7-19 and p. 22-28 of Reference (2) will be discussed at a later date. It is felt that the rigor of the treatment of this material might be in appropriate for the student at this time	2 ... 1 ...	20-21; 29-31 39-42 42, Ex. 1	N-(2) p. 3:3 (Draw graph); p. 6:2 N-(1) p. 47:1, 2 R-Selected review problems to provide remedial work for difficulties shown on test. p. 50-52

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
14	Properties of limits Note: Use intuitive concept of "bounded."	1 ... 42-45 2 ... 42, ex. 2 3 ... 31-33		N-(1)p. 47-48: 3, 5 N-(2) Write proof of Theorem 9(a) on p. 33 R-(2)p. 3:4; p. 6:4
15	Properties of limits (a) Prove $\lim_{n \rightarrow \infty} \left\{ \frac{n-1}{n} \right\} = 1$, n is a natural number. Also draw graph. (b) Prove $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$. Find relationship between ϵ, δ and Δx. Draw graph of function and label ϵ, δ, and Δx.	1 ... 45-48 2 ... 33-35		N-(1)p. 47-48: 4, 6, 7 N-(2) Write proof of Theorem 9(c) on p. 33 R-(2)p. 6:3

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
16	Properties of limits (Review for test)	1 ... 2 ...	39-48 1-6; 20-21; 29-30	<u>Review During Class</u> (1) Prove $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{10^n}\right) = 1$ n is a natural number. Also draw graph. (2) Prove $\lim_{x \rightarrow \frac{2}{3}} \frac{9x^2 - 4}{3x - 2} = 4$. Find relationship between ϵ and δ. Draw graph of function, and label ϵ, δ, and Δx. (3) Review theorems on limits
17	Test <u>Suggested Test</u> 1. Find and prove by definition the limit of sequence $\left\{1 + \frac{1}{n}\right\}$ where n is a natural number. Draw graph of sequence and label ϵ.			R-(1) p. 48:6, 7, 8 R-(2) Write proof of Theorem 9(b) on p. 33

LESSON	SCCPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
	<u>Suggested Test (cont'd)</u> 2. Find and prove by definition that the $\lim_{x \rightarrow \frac{4}{3}} \frac{9x^2 - 16}{3x - 4}.$ 30% Find the relationship between ϵ and δ Draw the graph of the function and label ϵ, δ, and Δx. 3. Prove the following: 30% If $L_1 = \lim_{t \rightarrow c} F_1(t)$ and $L_2 = \lim_{t \rightarrow c} F_2(t) \text{ both exist}$ and are finite, then $\lim_{t \rightarrow c} [F_1(t) + F_2(t)] = \lim_{t \rightarrow c} F_1(t) +$ $\lim_{t \rightarrow c} F_2(t) = L_1 + L_2.$ 4. Given: $\lim_{x \rightarrow 2} f(x) = 7$. 5% Find: $\lim_{x \rightarrow 2} \frac{\cos(x-2)}{f(x)}$		

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
	<u>Suggested Test (cont'd)</u> 5. May we have a zero difference between the number which is the limit of a function as $x \rightarrow a$ and the value of the function at $x = a$? 5%		

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
18	UNIT TWO DERIVATIVES OF ALGEBRAIC FUNCTIONS (Time allotment - 15 days) Polynomial functions and their derivatives (2-1) a. The derivative of a constant b. The derivative with respect to x, of x^n where n is a natural number c. The derivative of u, where $u = f(x)$ is a differentiable function of x, and c is a constant d. The derivative with respect to x of cx^n where n is a natural number e. The derivative of the sum of a finite number of differentiable functions is equal to the sum of their derivatives $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ f. The second derivative $y'' = \frac{dy'}{dx} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$	I ...	54-61	N-(1) p. 61: 1-15 odd R-(1) p. 52: 32 a, b Selected review problems to provide remedial work for difficulties shown on test

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
19	Velocity and acceleration a. Relation of velocity and first derivative b. Relation of acceleration and second derivative	1 ...	60	N-(1) p. 61:16 (3) p. 57:28 R-(1) p. 61:6, 10, 14, 17 (2) p. 33. Write proof of theorem 9d
20	Rational functions and their derivatives (2-2) a. The derivative of the product $y = uv$ of two differentiable functions of x is $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$	1 ...	61-63	N-(3) p. 56:12, 13, 14; p. 57:29 R-(1) p. 49:9, 10, 11 (2) p. 34. Write proof of Cor. 9.1
21	b. At a point where $v \neq 0$, the derivative of the quotient $y = \frac{u}{v}$ of two differentiable functions u and v is given by $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	1 ...	63-65	N-(3) p. 59:1a, c, f, 2; p. 57:32; p. 56:15 R-(1) p. 61:18; p. 49:12 (2) p. 35. Write proof of Cor. 9.2

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
22	c. If $u = g(x)$ is a differentiable function of x, and n is a positive integer or zero, then $\frac{d}{dx} \left(u^n \right) = n u^{n-1} \frac{du}{dx}$ d. At a point where $u = g(x)$ is differentiable and different from zero, the derivative of $y = u^n$ is given by $\frac{d(u^n)}{dx} = n u^{n-1} \frac{du}{dx} \quad \text{if } n \text{ is a negative integer.}$	1 ...	65, 66	N-(1) p. 67: 1-15 odd R-(3) p. 56: 16; p. 57: 31 (2) p. 35: Continuity [Draw graph of $f(x)$]
23	Implicit relations and their derivatives (2-3)	1 ...	68-72	N-(1) p. 72: 1, 3, 8, 11; p. 73: 30 R-(1) p. 68: 6, 8, 10 (2) p. 35. Write proof of theorem 10
24	Review and reinforcement of basic concepts in lessons 20-24			N-(1) p. 72: 4, 6, 14; p. 73: 24, 31 R-(1) p. 68: 12, 14, 16 (2) p. 40: 1-3

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
2	The increment of a function (2-4)	1 ...	73-77	N-(1) p. 77:2, 3, 5, 7 R-(1) p. 72:16; p. 73:26; p. 68:17 (2) p. 40:4, 5
26	The chain rule for derivatives (2-5)	1 ...	77-81	N-(1) p. 81:1, 3; p. 82:6, 7 R-(1) p. 77:8; p. 72:23; p. 73:32; p. 68:18 (2) p. 41:14, 15
27	The differentials dx and dy (2-6) Formulas for differentiation repeated in the notation of differentials (2-7). Parametric representations.	1 ...	82-88	N-(1) p. 88:1, 5, 7, 9; p. 89:14, 19 R-(1) p. 77:9; p. 72:19; p. 73:27; p. 82:8, 9 (2) p. 41:19

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
28	The differentials dx and dy	1 ... 86	N--(1) Draw graph of $y = x^2$. Draw tan line at $x = 1$ and label $dx, dy, \Delta x, \Delta y$. See p. 83 R--(1) p. 73:33; p. 82:10; p. 89: 15, 20; p. 88:6, 8, 11
29	Continuity (2-8)	1 ... 89-93	N--(1) Write proof of theorem 1 on p. 93; p. 97:1 R--(1) p. 73:28, 34; p. 89:16, 21; p. 88:10
30	Continuity (2-8)	1 ... 94-97	N--(1) Write proofs of theorems 2, 3i; p. 94-95; p. 97:2ab R--(1) p. 73:29; p. 88:12; p. 89:17

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
31	Review	1 ... 54-88	Discuss in Class <u>N-(1) p. 97:3, 4, 5, 8</u> R-(1) Selected problems p. 100-102
32	Test		R-(1) p. 97:6, 7; p. 98:5, 6, 7, 10

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT THREE APPLICATIONS (Time allotment - 14 days)			
33	Sign of the first derivative (3-1)	1 ...	104-105	N-(1) p. 106:1, 3, 5 R-(1) p. 100-101:38. Also selected review problems to provide remedial work for difficulties shown on test.
34	Related rates (3-2)	1 ...	106-110	N-(1) p. 110:1, 3, 4 R-(1) p. 100-101:42; p. 106:2 Also selected review problems to provide remedial work for difficulties shown on test.
35	Significance of the sign of the second derivative (3-3) Introduction to curve plotting (3-4) Maximum and minimum points Points of inflection Intercepts Extent	1 ... 1 ...	111-114 114-117	N-(1) p. 117-118:1, 4, 6, 7 R-(1) p. 101:51; p. 106:4 p. 110:5, 6

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
36	Curve plotting (3-4) Asymptotes Symmetry with respect to x-axis y-axis origin	1 ...	114-117	N-(1) p. 117-118:2, 3, 5, 10, 13 N-(3) p. 91:12 R-(1) p. 110:7, 8
37	Maxima and minima (Theory) (3-5)	1 ...	119-131	N-(1) p. 131:1, 2. Prove thm. p. 120 R-(1) p. 118:14, 17; p. 110:9 R-(3) p. 91:13
38	Maxima and minima: Problems (3-6)			N-(1) p. 131:4, 5 R-(1) p. 118:16, 19; p. 110:10 R-(3) p. 91:14

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
39	Maxima and minima: Problems (3-6)	1 ... 134-136	N-(1) p. 131:6; p. 132:8, 12 R-(1) p. 118:20; 110:11 R-(3) p. 91:16
40	Rolle's Theorem (3-7)	1 ... 136-141	N-(1) p. 136:1, 2 R-(1) p. 118:21; 132:9, 10; p. 110:12 R-(3) p. 91:15
41	The Mean Value Theorem (3-8)		N-(1) p. 141:1, 2 R-(1) p. 136:3; p. 132:11, 18; p. 111:13 R-(3) p. 91:17

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
42	Review and Extension of the Mean Value Theorem (3-9) Applications of the Mean Value Theorem to curve tracing	1 ... 141-144 1 ... 144-148		N-(1) p. 144:1; p. 148:1, 2 R-(1) p. 136:4; p. 111:14; p. 132:19; p. 141:3, 6 R-(3) p. 91:9
43	Theorems of Cauchy and L'Hospital Prove L'Hospital's Theorem for the form $\frac{0}{0}$ and then indicate that it may also be proved for the form $\frac{\infty}{\infty}$. Note: Mimeographed notes are available	3 ... 151-153		N-(2) p. 40:1-6 (Use L'Hospital's Theorem) R-(1) p. 149:8; p. 111:15; p. 133:20; p. 141:4, 8; p. 144:2; p. 148:8
44	Applications of Theorems in Lessons 40-43	1 ... 104-148		N-(2) p. 41:14, 15 (Use L'Hospital's Theorem) R-(1) p. 149:11, 17; p. 111:16 p. 133:21; p. 148:10 R-(3) p. 91:18

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
45	Review		<u>Discuss in Class</u> R-(1) p. 133:22 Selected problems (1) 148-151 (3) 91
46	Test		R-(1) p. 133:23-25

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
47	UNIT FOUR INTEGRATION (Time allotment - 24 days) Introduction (4-1) The indefinite integral (4-2)	1 ...	155-160	N-(1) p. 160-161: 1, 3, 7, 9, 14-18 R-(1) p. 133: 26 Selected review problems to provide remedial work for difficulties shown on test p. 148-151
		1 ...	155-160	
48	Applications of indefinite integration (4-3)	1 ...	161-163	N-(1) p. 163: 1, 3, 5, 7, 9 R-(1) p. 160-161: 2, 4, 5, 19-22; p. 133: 27
49	Area under a curve (4-6)	1 ...	181-186	N-(1) p. 186: 1, 2, 4 R-(1) p. 149: 19; p. 160-161: 6, 23; p. 163: 4, 10; p. 164: 13

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
50	Computation of areas as limits (4-7)	1 ...	186-191	N-(1) p. 191:2, 3 R-(1) p. 149:20; p. 161:10, 24; p. 164:14; p. 186:5
51	Areas by calculus (4-8)	1 ...	191-196	N-(1) p. 196-197:1, 3, 5, 11 R-(1) p. 149:21; p. 161:13, 25; p. 164:15; p. 191:4
52	The definite integral and the Fundamental Theorem of Calculus	1 ...	198-199	N-(1) Write proof of Fundamental Theorems R-(1) p. 164:16; p. 191:5; p. 196-197:2, 4, 6, 12

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
53	The definite integral and the Fundamental Theorem of Calculus	1 ...	199-201	N-(1) p. 205-206: 1ac, 6-9 R-(1) p. 197: 13, 14 p. 150: 33
54	The definite integral and the Fundamental Theorem of Calculus	1 ...	201-205	N-(1) p. 205-206: 1bd, 17, 19 R-(1) p. 197: 15; p. 150: 29
55	The trapezoidal rule for approximating an integral	1 ...	207-211	N-(1) p. 211: 1, 3 R-(1) p. 206: 21; p. 150: 30

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
56	Simpson Rule (7-11) Give algebraic rather than trigonometric illustration; for example: $\int_0^1 \frac{dx}{x^2 + 1}$	1 ...	385-387	N-(1) p. 388:1, 6 R-(1) p. 211:5; p. 220:48; p. 151:39
57	Review	1 ...	154-211 385-387	Selected Review Problems (3) p. 317: 1ab (1) p. 216-219
58	Test			N-(1) Study p. 211-215, Section 4-11 and Section 4-12; p. 216:1 (top), 2b

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
59	Introduction (5-1) Areas between two curves (5-2)	1 ...	222-224	N-(1) p. 224:1-3 R-(1) p. 216:3b (top) p. 150:25
60	Volume (5-4) - Method of "Slicing"	1 ...	229	N-(1) p. 239:14 N-(3) p. 303:13a R-(1) p. 224:4, 5, 6
61	Volume (5-4) - Method of Revolution	1 ...	230	N-(1) p. 239:1, 3, 11a R-(1) p. 224:7; p. 239:15 R-(3) p. 303:13b

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
62	Volume (5-4) - Method of Cylindrical Shells	1 ...	230-235	N-(1) p. 239:5 R-(1) p. 224:8; p. 239:4, 11b R-(3) p. 303:13
*63	Optional - Length of a plane curve (5-6)	1 ...	245	N-(1) p. 250:1, 2 R-(1) p. 224:10; p. 239:6, 11d, 12 R-(3) p. 14
*64	Optional - Length of plane curve (5-6) $s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \text{ (parametric form)}$			N-(1) p. 251:8 R-(1) p. 239:7, 9, 10, 11c, 13 p. 250:3

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
*65	Area of a surface of revolution (5-7)	1 ...	251-255	N-(1) p. 256:2 R-(1) p. 239:8, 11e, 17; p. 250:4; p. 251:9
66	Average value of a function (5-8)	1 ... 3 ...	257-259 326-327	N-(1) p. 259:3 N-(3) p. 327:1, 2 R-(1) p. 250:5; p. 156:3, 4
67	Distance (motion in a straight line)(5-3)	1 ...	224-228	N-(1) p. 228:1, 2 R-(1) p. 250:6; p. 256:5 R-(3) p. 328:3, 4; p. 299-1

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
68	Work	1 ... 277-282	N-(1) p. 282:1 N-(3) p. 321:1, 5 R-(1) p. 228:3, 4, 12 R-(3) p. 299:4; p. 290:14
69	Review	1 ... 154-266	<u>Do in class</u> R-(3) p. 321:2, 6, 8 R-(1) Selected problems p. 284-286
70	Test		R-(1) p. 164-168 (Study)

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT FIVE TRANSCENDENTAL FUNCTIONS (Time allotment - 16 days)			
71	Review of Trigonometry (4-4)	1 ...	164-171	R-(1) Study p. 169-171 Also selected review problems to provide remedial work for difficulties shown on test. (see p. 284-286)
72	Theorem (p. 172) and applications (p. 179)	1 ...	171-173, 179	N-(1) p. 179:1-8 R-(1) Selected review problems from p. 284-286
73	Differentiation of sines and cosines (4-5)	1 ...	173-177	N-(1) p. 180:22-36 even R-(1) p. 180:9-12 (Check by using L'Hospital's Theorem.)

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
74	Integration of sines and cosines (4-5)	1 ...	178-179	N-(1) p. 181:40-52 even R-(1) p. 180:13, 14 (check by using L'Hospital's Theorem.) p. 180:21, 23, 27, 29; p. 224:9
75	Differentiation of other trig functions (6-1)	1 ...	288-291	N-(1) p. 291:2, 13-23 odd R-(1) p. 181:41-47 odd p. 180:15 (check by L'Hospital) p. 180:31
76	Integration of other trig functions (6-1)	1 ...	290-291	N-(1) p. 292:40, 42, 44 R-(1) p. 291:14-22 even p. 291:4; p. 180-16 (check by L'Hospital) p. 181:49, 51

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
*77	Optional-Inverse functions (6-2) a. Algebraic - see Morrill b. Trigonometric	1 ... 3 ...	292-297 102-103	N-(3) p. 103:1a N-(1) p. 297:1; p. 298:3, 7a R-(1) p. 292:25, 41; p. 291:6, 24 p. 180:16, 35; p. 181:53, 54 p. 206:10
*78	Optional - Derivatives of the inverse trigonometric functions a) $\frac{d}{dx}(\arcsin u)$; b) $\frac{d}{dx}(\arccos u)$; c) $\frac{d}{dx}(\arctan U)$	1 ... 1 ...	298-300 298-300	N-(1) p. 300:5, 6 N-(3) p. 103:1c R-(1) p. 298:5; p. 292:38, 43, 45 p. 206:12, 13; p. 181:55, 57
*79	Derivative of the inverse functions a) $\frac{d}{dx}(\arccot u)$; b) $\frac{d}{dx}(\operatorname{arcsec} u)$; c) $\frac{d}{dx}(\operatorname{arccsc} u)$	1 ...	298-300	N-(1) p. 300:7, 8, 9 N-(3) p. 103:1d; p. 124:1 R-(1) p. 298:4; p. 292:48; p. 181:58; p. 180:17

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
80	The natural logarithm (6-4)	1 ...	301-302	N-(1) p. 302:1 R-(1) p. 300:10, 11, 16; p. 298:7c p. 292:49 R-(3) p. 103:1e; p. 124:2
81	The derivative of $\ln x$ (6-5)	1 ...	303-305	N-(1) p. 305:1-11 odd R-(1) p. 302:2; p. 300:13, 17; p. 298:7b; p. 292:34; p. 181:56, 58 R-(3) p. 119:1
82	Properties of natural logarithms (6-6) The graph of $y = \ln x$ (6-7)	1 ...	308-310	N-(1) p. 308:1, 2, 3; p. 310:1 R-(1) p. 305:2-12 even p. 306:19, 20, 21 R-(3) p. 103:3, 4

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
83	The exponential function (6-8)	1 ...	311-316	N-(1) p. 316: 1abc, 2, 3, 4, 22, 23 R-(1) p. 306: 13, 14, 22, 23; p. 308: 4 R-(3) p. 119: 2
84	The function a^u (6-9) The function $\log_a u$ (6-10)	1 ... 1 ...	317-322 323-324	N-(1) p. 322: 1, 2, 3, 12 R-(1) p. 316: 1 def. 5, 6, 24, 25; p. 306: 15, 24, 25; p. 308: 5; p. 310: 3
85	Review	1 ...	288-324 510-514	Discuss in Class R-(1) p. 322: 4, 5, 13, 14 p. 316: 1gh, 8, 13, 26, 27 p. 306: 16, 26; p. 300: 18 p. 300: 18; p. 180: 18, 25, 59

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
86	Test		N-(1) Study p. 325-329 Do p. 329:2

LESSON	SCCPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
	UNIT SIX TECHNIQUES OF INTEGRATION (PART I) (Time allotment - 14 days)		
87	Basic Formulas (7-1)	1 ... 337-339	N-(1) p. 340: 1-13 odd R-(1) Selected review problems to provide remedial work for difficulties shown on test p. 331-335
88	Powers of trigonometric functions (7-2) $\int \sec x \, dx = \ln \sec x + \tan x + c$ $\int \csc x \, dx = \ln \csc x - \cot x + c$	1 ... 341-346	N-(1) p. 346: 1, 3, 5, 7a, 9, 11 R-(1) Selected review problems p. 331-335; p. 340: 15-23 odd
89	Even powers sines and cosines (7-3) (stress formulas 2(a), 2(b), p. 348)	1 ... 347-349	N-(1) p. 349: 1, 3, 4 R-(1) p. 340: 25, 27, 29 p. 346: 13, 15, 17, 19

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
90	Integrals involving $\sqrt{a^2 - u^2}$, $\sqrt{a^2 + u^2}$, $\sqrt{u^2 - a^2}$, $a^2 + u^2$, and $a^2 - u^2$ (stress formulas (4) and (5), p. 350)	1 ... 349 - 355		N-(1) p. 355:1, 3, 5, 9 R-(1) p. 340:31, 33; p. 346:21; p. 349:5
91	Integrals involving $ax^2 + bx + c$ (7-5)	1 ... 356-359		N-(1) p. 359:1, 3 R-(1) p. 340:35, 37; p. 347:23, 25; 349:6; p. 355:2, 4, 10
92	Integration by partial fractions (7-6)- Example 1	1 ... 359-360		N-(1) p. 364:1, 3 R-(1) p. 340:14; p. 347:27; p. 349:7, 8; p. 355:7; p. 359:2

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
93	Integration by partial fractions (7-6) (Examples 2, 3, 4 - p. 361)	1 ...	361-364	N-(1) p. 364:5, 7 R-(1) p. 349:9; p. 356:13; p. 359:4; p. 347:24
94	Integration by parts (7-7)	1 ...	365-369	N-(1) p. 369:1, 5 R-(1) p. 349:10; p. 356:18; p. 359:5; p. 364:7
*95	Optional-Integration of rational functions of $\sin x$ and $\cos x$, and other trigonometric integrals	1 ...	370-373	N-(1) p. 373:1 R-(1) p. 356:16; p. 359:7; p. 369:7, 8; p. 364:6

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
*96	Optional - Further substitutions (7-9)	1 ...	374-377	N-(1) p. 377:1, 3 R-(1) p. 356:19; p. 359:10; p. 369:6; p. 373:2; p. 364:8
97	Improper integrals and convergence	1 ...	378-384	N-(1) p. 384:1, 3 R-(1) p. 359:8; p. 369:12; p. 365:9; p. 373:5; 377:5
98	Numerical methods for approximating definite integrals; Simpson's Rule-(7-11)	1 ...	385-387 359. ex. 1, 3, 6, 9	N-(1) p. 388:2 R-(1) p. 359:6; p. 365:10; p. 370:13; p. 373:6; p. 384:5; 377:7

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
99	Review	I . . . 337-389	Selected Review Problems p. 389-392
100	Test		Selected Review Problems p. 331-33

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT SEVEN POLAR COORDINATES (Time allotment - 9 days)			
101	The polar coordinate system (11-1)	1 ...	535-539	N-(1) p. 539: 1b, 2abc, 3a, 4a, 9 R-(1) Selected review problems, p. 389-392
102	Graphs of polar equations (11-2)	1 ...	539-543	N-(1) p. 543: 1, 3 R-(1) Selected review problems, p. 389-392 p. 539: 1c, 2dg, 3c, 4b, 12
103	Polar equations of the conic sections and other curves (11-3)	1 ...	544-548	N-(1) p. 549: 3, 8, 14a R-(1) p. 543: 2, 5; p. 539: 5, 10

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
104	Plane arcs in polar coordinates (11-5)	1 ...	555-557	N-(1) p. 557:1, 3 R-(1) p. 543:6; p. 549:4, 10, 14d
105	Parametric equations in analytic geometry	1 ...	565-571	N-(1) p. 571:1, 3 R-(1) p. 543:11; 557:4; p. 559:13a
106	Differential equations (6-11)	1 ...	325-328	N-(1) p. 329:3, 4 R-(1) p. 558:5; p. 559:9; p. 571:4, 5

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
107	Differential equations (6-11)	I ... 325-328	N-(1) p. 329:1, 6 R-(1) p. 558:6: 559:2; p. 571:7
108	Review	I ... 535-557 325-328	Selected Review Problems p. 329, 539, 543, 549, 557, 559, 571
109	Test		

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT EIGHT APPLICATIONS (Time allotment - 15 days)			
110	The exponential function and vertical projectiles	1 ... 325-329 3 ... 174-176		N-(1) p. 332:53; p. 333:62 N-(3) p. 176:1, 2; p. 480:1
111	The exponential function	3 ... 169-171 Morrill		N-(3) p. 174:11 R-(1) p. 333:69, 70 R-(3) p. 176:3; p. 480:2
112	Work	1 ... 277-282 3 ... 318-321		N-(1) p. 282:4 R-(1) p. 333:65, 67 N-(3) p. 321:6, 8; p. 176:4

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
113	Average value of a function	1 ...	257-259	N-(1) p. 259: 1ab R-(1) p. 334: 81 R-(3) p. 176: 5; p. 322: 9; p. 480: 3
114	Moments and center of mass	1 ...	259-266	N-(1) p. 266: 1, 2 R-(1) p. 259: 2a, 6; p. 334: 82 R-(1) p. 322: 9
115	Moments and center of mass	1 ...	259-266	N-(1) p. 266: 3, 4, 5, 6 R-(3) p. 176: 6; p. 322: 10

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
116	Review of lessons 110-115		R-(1) p. 266: 5, 7; p. 558: 6 bottom; p. 259: 7 p. 330: 4, 5, 9 R-(3) p. 480: 4
117	Centroid and center of gravity	1 ... 267-269	N-(1) p. 270: 1, 2, 3 R-(1) p. 266: 8, 9; p. 259: 8 R-(3) p. 480: 5
118	Theorems of Pappus	1 ... 270-273	N-(1) p. 273: 1, 2 R-(1) p. 270: 4, 6; p. 266: 10; p. 259: 9

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
119	Hydrostatic Pressure	1 ...	274-277	N-(1) p. 277:1, 2 R-(1) p. 266:11; p. 270:7; p. 273:3, 4
120	Non-vertical Projectiles	3 ...	Morrill (9.4)	N-(1) p. 564:1 R-(1) p. 270:8; p. 273:5; p. 277:3, 4 R-(3) p. 481:6
121	Non-vertical projectiles	1 ... 3 ...	561-564 Morrill (9,4)	N-(1) p. 564:2, 3 R-(1) p. 270:9; p. 273:6 p. 277:5

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
122	Cumulative review - lessons 117-121		R-(1) p. 564:4; p. 270:10; p. 277:6; p. 273:7; p. 282:5
123	Review of lessons 110-122		R-(1) p. 283:3, 6-9; p. 277:7; p. 273:8; p. 270:11 R-(3) p. 182:1, 2; p. 481:7
124	Test		(1) Selected Problems p. 284-287 (3) p. 182

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT NINE PLANE ANALYTIC GEOMETRY (Time allotment - 16 days)			
125	Curves and equations (9-1)	1 . . .	437-444	N-(1) p. 444: 1, 2, 8 R-(1) Selected problems from p. 284-286
126	Curves and equations (9-1)	1 . . .	437-444	N-(1) p. 444: 3, 4, 10 R-(1) Selected problems from p. 284-286
127	Tangents and normals (9-2)	1 . . .	444-449	N-(1) p. 449: 1, 2, 5, 7 R-(1) p. 444: 5, 6

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
128	Newton's method for approximating roots of equations	1 ...	451-455	N-(1) p. 455:1, 3 R-(1) p. 444:9; p. 449:3; p. 450:9
129	Distance between two points (9-4) The circle (9-5); include parametric form	1 ... 1 ... 1 ...	457-458 459-463 565	N-p. 464:3, 9, 13; p. 458:2, 5 R-p. 444:7; p. 450:8; p. 455:4
130	Determinants (8-6)			N-p. 425:1 R-p. 450:11; p. 455:5; p. 458:3, 6; p. 464:5, 11, 15, 16

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
131	Application of determinants to the circle determined by 3 conditions (9-5)	1 ...	459-464	N-(1) p. 464:17 R-(1) p. 425:4; p. 450:13; p. 455:6; p. 458:7; p. 464:10, 14
132	The parabola (9-6)	1 ...	465-471 Morrill 107, 108	N-(1) p. 471:4, 8; p. 472:14 N-(3) p. 108:2a (sketch curve) R-(1) p. 425:3; p. 450:15; p. 459:8; p. 464:19
133	The ellipse (9-7)	1 ...	473-481 Morrill	N-(1) p. 481:4, 8b, 9 R-(1) p. 471:6, 11; p. 472:19; p. 450:21a R-(3) p. 108:2b

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
134	The ellipse (9-7); parametric form	Morrill	p. 107; p. 289, Ex. 5	N-(3) p. 211: 1d, 10 R-(1) p. 482: 10 R-(3) p. 310: 6; p. 302: 3
135	The hyperbola (9-8) (parametric form)	1 ... 482-490 Morrill	p. 107	N-(1) p. 490: 1b, 3 R-(3) p. 303: 7 R-(1) p. 481: 6, 8c; p. 472: 21, 24
136	Rotation of second degree curves (9-9)	1 ... 490-494		N-(1) p. 494: 1, 2 R-(1) p. 472: 28a; p. 482: 12; p. 490: 10

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
137	Rotation of second degree curves	1 ...	490-494	N-(1) p. 495: 3, 8 R-(1) p. 472: 27; 482: 11; p. 490: 13 R-(3) p. 310: 8f
138	Invariants and the discriminant	1 ...	495-497	N-(1) p. 497: 1, 2, 3, 4 R-(1) p. 495: 6, 9; p. 490: 4; p. 482: 15
139	Review	1 ...	437-494	Selected review problems p. 437-494

LESSON	SCOPE	REFERENCES CODE PAGES	SUGGESTED ASSIGNMENTS
140	Test		N-Study p. 573-577
141- 150	Review for CEEB Advanced Placement Examination		See mimeographed review for CEEB Lessons and Problems

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
	UNIT TEN (OPTIONAL) VECTORS AND PARAMETRIC EQUATIONS (Time allotment -5 days)			
151	Vector components (12-3)	1 ... 573-577		N-(1) p. 577-578: 1-11 odd R-(1) Selected review problems to provide remedial work for difficulties indicated on test
152	Differentiation of vectors (12-4)	1 ... 578-582		N-(1) p. 582: 1, 3, 5, 8 R-(1) p. 577-578: 2-12 even R-(1) Selected review problems to provide remedial work for difficulties indicated on test
153	Tangential vectors (12-5)	1 ... 583		N-(1) p. 585: 1, 2 R-(1) p. 582: 2, 4, 6; p. 578: 13, 18

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
154	Tangential and normal components of the	1 ...	591-595	N-(1) p. 594: 1, 3 R-(1) p. 578: 14; p. 582: 9; p. 585: 3, 4
155	Polar coordinates (12-8)	1 ...	595-597	N-(1) p. 597: 1, 2 R-(1) p. 578: 15; 583: 10; p. 585: 5; p. 594: 5

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
156	UNIT ELEVEN (OPTIONAL)			
	SOLID GEOMETRY AND VECTORS (Time allotment - 10 days)			
	Space coordinates (13-1)	1 ... 603-605		N-(1) p. 606: 3, 5, 9, 13 R-(1) p. 594: 6; p. 597: 3, 4; p. 599: 2
157	Vectors (13-2)	1 ... 606-608		N-(1) p. 608: 3, 5, 6a, 7 R-(1) p. 606: 4, 6, 10, 16; p. 597: 5
158	Scalar product of two vectors (13-3)	1 ... 609-614		N-(1) p. 614: 1, 2, 3 (angle A only) (1) p. 598: 1-4; p. 606: 7, 11, 15; p. 608: 2, 6b, 8

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
159	The vector product of two vectors (13-4)	1 ...	615-619	N-(1) p. 619:1, 2 R-(1) p. 606:8, 12, 16; p. 608:6, 9; p. 614:3 (angle B only); p. 598:5
160	Equations of lines and planes (13-5)	1 ...	620-626	N-(1) p. 626:1, 2 R-(1) p. 619:3; p. 620:4; p. 606:17, 19; p. 608:11; p. 598:6; p. 614:4
161	Products of three or more vectors (13-6)	1 ...	628-633	N-(1) p. 634:1, 2 R-(1) p. 620:5; p. 626:3; p. 599:3; p. 609:12; p. 614:5

LESSON	SCOPE	REFERENCES		SUGGESTED ASSIGNMENTS
		CODE	PAGES	
162	Differentiation of vectors and of their products (13-7)	1 ...	634-637	N-(1) p. 637:1 R-(1) p. 620:6; p. 626:4; p. 627:8, 11; p. 634:3, 10
163	Space curves, curvature, and arc length	1 ...	637-641	N-(1) p. 641:1, 2 R-(1) p. 620:7; p. 626:5, 7; p. 627:12; p. 634:4, 7; p. 637:3
164	Review	1 ...	603-641 573-598	Selected Review Problems
165	Test			